On Foundations of Insurance Mathematics

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History of Insurance in Europe

- Prehistory
- Maritime insurance
- Fire Insurance
- Life Insurance
  - Price of annuities and other types of life insurance
  - Premium calculation
- Pension systems
Prehistory

- Early methods of transferring or distributing risk:
  - Chinese merchants travelling treacherous river rapids would redistribute their wares across many vessels to limit the loss due to any single vessel's capsizing (3rd millennium BC)
  - The Greeks and Romans introduced the origins of health and life insurance c. 600 AD when they organized guilds called "benevolent societies" which cared for the families and paid funeral expenses of members upon death.
  - Guilds in the Middle Ages served a similar purpose
Maritime Insurance

• In the late 1680s, Mr. Edward Lloyd opened a coffee house that became a popular haunt of ship owners, merchants, and ships’ captains, and thereby a reliable source of the latest shipping news

• It became the meeting place for parties wishing to insure cargoes and ships, and those willing to underwrite such ventures
Fire Insurance

- Insurance as we know it today can be traced to the Great Fire of London, which in 1666 devoured 13,200 houses.
- In the aftermath of this disaster, Nicholas Barbon opened an office to insure buildings. In 1680, he established England's first fire insurance company, "The Fire Office," to insure brick and frame homes.
Life Insurance

• The concept of Annuity – a yearly salary paid till one’s death was elaborated in early 1690s by a mathematician and astronomer Edmund Halley.

• He published in Philosophical Transactions of the Royal Society an article *An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslaw; with an Attempt to ascertain the Price of Annuities upon Lives*. 
Halley’s article

• Halley underlined the importance of the proper study of people’s mortality

• We read in his article:

THE Contemplation of the Mortality of Mankind [...] have been [...] considered by [...] Sir William Petty, in his Natural and Political Observations on the Bills of Mortality of London [...] But the Deduction from those Bills of Mortality seemed even to their Authors to be defective: First, In that the Number of the People was wanting. Secondly, That the Ages of the People dying was not to be had. And Lastly, That both London and Dublin by reason of the great an casual Accession of Strangers who die therein, (as appeared in both, by the great Excess of the Funerals above the Births) rendered them incapable of being Standards for this purpose
Halley’s article, continued

• We further read:
  – This *Defect seems in a great measure to be satisfied by the late curious Tables of the Bills of Mortality at the City of Breslaw [...]* wherein both the Ages and Sexes of all that die are monthly delivered, and compared wiht the number of the Births, for Five Years last past, viz. 1687, 88, 89, 90, 91 [...] 
  – This City of Breslaw is the Capital City of the Province of Silesia; near the Confines of Germany and Poland [...] It is very far from the Sea [...] confluence of Strangers is but small, and the Manufacture of Linnen employs chiefly the poor People of the place, as well as of the Country round about
Life tables, observations

• I will suppose the People of Breslaw to be increased by 1238 Births annually.
• Of these it appears by the same Tables, that 348 do die yearly in the first Year of their Age,
• and that but 890 do arrive at a full Years Age;
• and likewise, that 198 do die in the Five Years between 1 and 6 compleat, taken at a Medium;
• so that but 692 of the Persons born do survive Six whole Years.
Life tables of Breslaw population - numbers

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<td>84</td>
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Calculation of the price of Annuity

• Question: what price shall pay a person of age x who wants to buy a Life Annuity of EUR10000, i.e. yearly payment of EUR10000 till his/her death since the beginning of the next year of his/her life?

• We need to know the EXPECTED VALUE OF THE MONEY PAID FOR SUCH A PERSON
Calculation of the price of Annuity, continued

• Let \( l_y \) be the number of persons of the same sex who were among say 1000 persons born \( y \) years ago and are still alive at their age of \( y \)

• Let \( i \) denote the yearly interest rate (i.e. the present value of 1 EURO which we will obtain after \( n \) years equals \( (1+i)^{-n} \))

• The price life annuity equals

\[
a_x = 10000 \left( (1+i)^{-1} \frac{l_{x+1}}{l_x} + (1+i)^{-2} \frac{l_{x+2}}{l_x} + .... \right)
\]
Other types of insurance

- Finite Horizon Annuity: yearly payment of say EUR10000 since the beginning of the next year of one’s life for next n years or till his/her death

- The price of such annuity equals

\[
\begin{align*}
a_{x:n} &= 10000 \left( (1+i)^{-1} l_{x+1} + (1+i)^{-2} l_{x+2} + \ldots + (1+i)^{-n} l_{x+n} \right) / l_x
\end{align*}
\]

- Life Insurance - single payment of say EUR10000 at the moment of one’s death

\[
\begin{align*}
A_{x:\infty}^1 &= 10000 \left( (1+i)^{-1} (l_{x+1} - l_{x+2}) + (1+i)^{-2} (l_{x+2} - l_{x+3}) + \ldots \right) / l_x
\end{align*}
\]
Premium calculation

• Instead of paying once, annuity and life insurance are may be paid with several payments.

• Let us assume that a person at age x wants to pay yearly (since now) a premium P in order to get life annuity of EUR10000 at the age x+n. We have

\[
P = 10000 \cdot \frac{(1+i)^{-n}}{1 + a_{x+n}}
\]
Pensions across Europe – history and current regulation

• Great Britain
  – 1670s First organized pension scheme for Royal Navy Officers
  – 1908 Old Age Pensions Act - introduced first general old age pension paying a non-contributory amount of between 10p and 25p a week, from age 70, on a means-tested basis from January 1 1909 - "Pensions Day". This was introduced during the Liberal government of David Lloyd-George. Sir William Beveridge, father of the welfare state, was an adviser.
Pensions across Europe – history and current regulation

– **1925** Contributory Pensions Act - set up a contributory State scheme for manual workers and others earning up to £250 a year. The pension was 50p a week from age 65.

– **1946** National Insurance Act - introduced contributory State pension for all. Initially pensions were £1.30 a week for a single person and £2.10 for a married couple. Paid from age 65 for men and 60 for women, effective from 1948.
Pensions across Europe – history and current regulation

– **1959** National Insurance Act - introduced a top-up state pensions scheme, based on earnings and known as the graduated pension. Covered earnings between £9 and £15 a week.

– **1975** Social Security Pensions Act - set up the State Earnings related Pension Scheme (Serps). Introduced in 1978, the scheme replaced graduated pensions. Rules for contracting out were also introduced, whereby workers with adequate private provision can give up all or part of the benefits of Serps. In return they pay lower National Insurance contributions.
Pensions across Europe – history and current regulation

– **1980** Social Security Act - Link between state pension increases and average earnings broken by Margaret Thatcher's Conservative government. If the link with earnings had not been broken, a basic state pension for a single pensioner would worth about £30 a week more.

– **1986** Financial Services Act - set out terms and conditions under which investment business could be conducted. Changes to contracting out. Taxation of pension fund "surpluses" introduced.
Pensions across Europe – history and current regulation

• Germany
  – Bismarck advised Emperor Wilhelm II to establish social Insurance
  – 1891 old-age pension plans if one was 70 years old (average life expectancy was 45 years at the time)
  – 1911 The Imperial Insurance Code signaled the next step towards a modern pension scheme. From that point on, it was also for example lawfully provided that the bereaved were looked after by the state if they could not sustain themselves after the death of their spouse
Pensions across Europe – history and current regulation

– **1957** it was here that the pay-as-you-go scheme that is so typical for Germany was introduced, as well as a pension formula that calculates the earnings during old age based on the obtained earnings during the time in gainful employment.

– **1970s** the retirement age was shaped more flexibly and now, self-employed people, students and housewives could profit of the lawful pension insurance.
Problems

• Aging society ...
Insurance market worldwide

• Insurance premiums worldwide (in mln EUR)

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<tr>
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<tr>
<td>Japan</td>
<td>249 294</td>
<td>498 452</td>
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<tr>
<td>Rest of the World</td>
<td>146 625</td>
<td>352 990</td>
<td>364 878</td>
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Source: World Insurance in 2003, Swiss Re

Life insurance – about 40%-60% in EU
Non life insurance (e.g. car insurance)