Financial Econometrics II
Credit Risk. Part 1.

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Basics
Credit risk definition

„The risk that a borrower or counterparty may fail to fulfill an obligation.”

The assessment of credit risk:
- Evaluating PD (probability of default) by the counterparty, obligor, or issuer
- Evaluating exposure or financial impact on the institution in the event of default.
Credit risk regulations. Basel Committee.

- Basel Committee of Banking Supervision (BCBS) is housed in the Bank of International Settlements (BIS) in Basel, Switzerland.
- Created in 1974 by the central bank governors (G10 countries)
- BCBS now composed of around 30 countries
- 1980 – set of standards on setting minimum capital requirements for banks (Basel I)
- 2010 – response to global financial crisis (GFC) known as Basel III.
- Now ongoing Basel IV...
Definitions of key components of credit portfolio analysis

- **Probability of default (PD):** The probability that an obligor will not meet a stated obligation. In the case of a loan, the obligor is the borrower and the obligation is to pay a regular coupon and repay the principal at maturity.

- **Loss Given Default (LGD):** The amount lost when an obligor ails to meet a stated obligation. Presented in percent. 1-LGD is a recovery.

- **Time horizon of analysis (H):** Time horizon over which analysis is made. Mostly is one year, but last changes in regulations (IFRS9) motivate institutions to use longer periods (lifetime approach).

- **Default correlation:** The correlated movement into default of two obligors

- **Value correlation:** The correlated movement in the value of the credit-risky securities within a portfolio
Definitions of key components of credit portfolio analysis

• **Exposure at Default (EAD)**: the gross exposure under a facility upon default of an obligor.

• **Expected Loss**: 

\[ \text{EL} = \text{PD (in%)} \times \text{LGD (in%)} \times \text{EAD (in $)} \]

(over time horizon H)
Definitions of key components of credit portfolio analysis

Capital adequacy - general idea:

\[ \text{CAR} = \frac{\text{capital}}{\text{RWA}} \]

CAR – capital adequacy ratio
RWA – risk weighted assets

- CAR is higher than simple capital assets ratio
- Basel I: Tier 1 and Tier 2 capital, CAR >= 8%
- Basel Amendment 1996 – Tier 3 introduced
- Basel II: CAR >= 8%
- Basel III: additional capital buffers introduced
Definitions of key components of credit portfolio analysis

Two approaches to risk calculations:
- Standardised Approach
- Internal Model Approach

To be eligible to adopt the IRB approach, banks must pass the use test of its national regulator.

Under IRB approach, banks must divide their banking book exposures into six broad classes of assets: corporate, sovereign, bank retail, equity and “other” exposures.
Definitions of key components of credit portfolio analysis

Basel II’s Three Pillar Approach:

• Pillar 1: Risk pillar
• Pillar 2: Supervisory pillar
• Pillar 3: Market discipline pillar
Calculating risk weights under foundation IRB

\[ E[L] = PD \cdot EAD \cdot LGD \]

\[ E[L|X = \alpha] = P(D = 1|X = \alpha) \cdot E[L|D = 1, X = \alpha] \]
\[ = CPD \cdot CLGD \]

CPD – Conditional PD, CLGD – Conditional LGD (can be interpreted as „downturn LGD”), X – systemic risk factor, L – loss rate on exposure, D – indicator variable (D=1 is default, 0 otherwise)

Financial institutions must hold for each exposure \( E[L|X = \alpha] \). It is percentage of provisions and capital to satisfy 99.9% VaR target (\( \alpha – 99.9\text{th percentile of standard normal distribution} \))
Calculating risk weights under foundation IRB

- Banks can determine their own estimation for some components of risk measure: the probability of default (PD), loss given default (LGD), exposure at default (EAD) and effective maturity (M).
- For public companies, default probabilities are commonly estimated using either the structural model of credit risk proposed by Robert Merton (1974) or reduced form models.
- For retail and unlisted company exposures, default probabilities are estimated using credit scoring or logistic regression, both of which are closely linked to the reduced form approach.
- For other portfolios there are hybrid methods, often adjusted to specifics.
Calculating risk weights under foundation IRB

The goal

→ to define risk weights by determining the cut-off points between and within areas of the expected loss (EL) and the unexpected loss (UL), where the regulatory capital should be held, in the probability of default.

→ Then, the risk weights for individual exposures are calculated based on the function provided by Basel II.
Calculating risk weights under foundation IRB

In the IRB approach, financial institutions are not required to estimate CPDs directly. Instead, a mapping function was provided by BCBS to calculate an exposure’s conditional default probability from their own estimate of expected default probability.

\[
CPD = \Phi \left( \frac{\Phi^{-1}(PD) + \sqrt{\rho} \cdot \Phi^{-1}(0.999)}{\sqrt{1 - \rho}} \right)
\]
Calculating risk weights under foundation IRB: corporate exposures

1) Correlation

\[
R = AVC \cdot \left( 0.12 \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} + 0.24 \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} \right)
\]

AVC (Asset Value Correlation) was introduced by the Basel III Framework, and is applied as following:

- If the company is a large regulated financial institution (total asset equal or greater to US $100 billion) or an unregulated financial institution regardless of size, \( AVC = 1.25 \)
- Else, \( AVC = 1 \)

2) Maturity adjustment

\[
b = (0.11852 - 0.05478 \cdot \ln(PD))^2
\]

3) Capital requirement

\[
K = LGD \cdot \left[ N \left( \sqrt{\frac{1}{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0.999) \right) - PD \right] \cdot \frac{1 + (M - 2.5)b}{1 - 1.5b}
\]

4) Risk-weighted assets

\[
RWA = K \cdot 12.5 \cdot EAD
\]

For SME special adjustment!

BCBS notations: \( N() \) – CDF of normal distribution, \( G() \) – inverse CDF of normal distribution, \( M \) – maturity in years
Calculating risk weights under foundation IRB: residential mortgage exposures

1) Correlation

\[ R = 0.15 \]

2) Capital requirement

\[ K = LGD \cdot \left[ N \left( \sqrt{\frac{1}{1 - R}} \cdot G(PD) + \sqrt{\frac{R}{1 - R}} \cdot G(0.999) \right) - PD \right] \]

3) Risk-weighted assets

\[ RWA = K \cdot 12.5 \cdot EAD \]

BCBS notations: N() – CDF of normal distribution, G() – inverse CDF of normal distribution
Calculating risk weights under foundation IRB: unsecured retail exposure

1) Correlation
   \[ R = 0.04 \]

2) Capital requirement
   \[
   K = LGD \cdot \left[ N\left( \sqrt{\frac{1}{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0.999) \right) - PD \right]
   \]

3) Risk-weighted assets
   \[
   RWA = K \cdot 12.5 \cdot EAD
   \]

BCBS notations: N() – CDF of normal distribution, G() – inverse CDF of normal distribution
Exercise
Calculate risk weights under foundation IRB: exercise 1

Calculate risk weights for:

a) Corporate exposure of company A (telecom), B (regulated insurance company), C (commercial bank)
b) Mortgage retail exposure of client Y
c) Credit card exposure of client Z

(data are available in exercise1.xls)
Ratings
Agency ratings

- Agencies have rated bond issues for over a century.
- S&P, Moody's, Fitch are the largest agencies.
- Until recently, these were single-name issues, mainly corporate, sovereign and municipal bonds.
- Corporate ratings are ordinal and express qualitative judgment of creditworthiness.
- Mapping to default probabilities (PD) is a recent adaptation to modeling applications - first studies of ratings performance in 1990s.
- Ratings are long-term and "through-the-cycle." Sticky by intent.
- Ratings of structured products (e.g., CDOs, RMBS) use same letter scale, but are quite different in methodology. Model-driven.
<table>
<thead>
<tr>
<th>Moody's</th>
<th>S&amp;P</th>
<th>Fitch</th>
<th>Rating Description</th>
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<tr>
<td>Long-term</td>
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<td>Aaa</td>
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Agency rating scales

To explain, we will employ S&P notation for long-term ratings.

- **Whole-letter rating categories** aggregate across + / - qualifiers, e.g., AA+, AA, AA- into AA.
- **Notches** refer to steps on the finer scale, so a move from AA+ to AA- is a two-notch downgrade.
## Agency ratings

Default experience of rated corporate bonds

### Historical Default Experience of Bonds Rated by Fitch

#### Investment-grade Bonds

<table>
<thead>
<tr>
<th>Rating at issuance</th>
<th>AAA</th>
<th>AA+</th>
<th>AA</th>
<th>AA−</th>
<th>A+</th>
<th>A</th>
<th>A−</th>
<th>BBB+</th>
<th>BBB</th>
<th>BBB−</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-year default probability</td>
<td>0.19%</td>
<td>0.57%</td>
<td>0.89%</td>
<td>1.15%</td>
<td>1.65%</td>
<td>1.85%</td>
<td>2.44%</td>
<td>3.13%</td>
<td>3.74%</td>
<td>7.26%</td>
</tr>
<tr>
<td>Default rate (annualized)</td>
<td>0.02%</td>
<td>0.06%</td>
<td>0.09%</td>
<td>0.12%</td>
<td>0.17%</td>
<td>0.19%</td>
<td>0.25%</td>
<td>0.32%</td>
<td>0.38%</td>
<td>0.75%</td>
</tr>
</tbody>
</table>

#### Speculative-grade Bonds

<table>
<thead>
<tr>
<th>Rating at issuance</th>
<th>BB+</th>
<th>BB</th>
<th>BB−</th>
<th>B+</th>
<th>B</th>
<th>B−</th>
<th>CCC+</th>
<th>CCC</th>
<th>CC</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-year default probability</td>
<td>10.18%</td>
<td>13.53%</td>
<td>18.46%</td>
<td>22.84%</td>
<td>27.67%</td>
<td>34.98%</td>
<td>43.36%</td>
<td>48.52%</td>
<td>77.00%</td>
<td>95.00%</td>
</tr>
<tr>
<td>Default rate (annualized)</td>
<td>1.07%</td>
<td>1.45%</td>
<td>2.04%</td>
<td>2.59%</td>
<td>3.24%</td>
<td>4.30%</td>
<td>5.68%</td>
<td>6.64%</td>
<td>14.70%</td>
<td>29.96%</td>
</tr>
</tbody>
</table>
Bank ratings

Basic rules:

• Rating/scoring models estimates PD for each exposure. So each exposure has its own PD.
• Rating class has the same level of risk (same PD)
• The worser the rating, the risk is higher
• **Rating system should have at least 7 rating classes for non-defaulted clients/exposures and at least one class for defaulted clients.**
• The rating model assignes loans to rating classes and estimates PD using information about:
  a) Client individual factors: qualitative and quantitative (for example marital status, ROA, LTV...)
  b) Macroeconomic factors (GDP, interest rates)
Bank ratings

Basic rules:

• Default definition can be assigned to client or to loan. Frequent approach in practice is assigning the rating to the client. So if one loan of particular client is past due, rating is getting worser for all bank exposures toward this client.

• Sometimes ratings are transferred from the country (sovereign) or parent company.

• Each bank have its own definition of default, however it must comply with Financial Supervisory recommendations.

• Basic criterion of classification to default class is 90DPD – 90 days past due.
Models of risk parameters: PD models
Dealing with outliers

Data used for risk parameters calculations often contain a few extreme values. They can reflect genuinely exceptional situations of borrowers, but they can also be due to data errors.

In any case extreme values can have a large influence on coefficient estimates, which could impair the overall quality of the scoring model.

What to do then?

1. Examine the distribution of the variables. Good indicator for the existence of outliers is for example the excess kurtosis (for normal distribution equal to zero)
2. Use winsorization technique
Dealing with outliers - winsorization

See example:

01_winsorizing.R
A score summarizes the information contained in factors that affect default probability. Standard scoring models take the most straightforward approach by linearly combining those factors. Let $x$ denote the factors (their numer is $K$) and $b$ are the weights (coefficients):

$$Score_i = b_1x_{i1} + b_2x_{i2} + \ldots + b_Kx_{iK} = b'x_i$$

The scoring model should predict default probability. We need to link scores with default probabilities as a function $F$ of scores:

$$Prob(\text{Default}_i) = Prob(y_i = 1) = F(Score_i)$$

Logistic distribution function $\Lambda(z) = \frac{\exp(z)}{1 + \exp(z)}$ applied:

$$Prob(\text{Default}_i) = \Lambda(Score_i) = \frac{\exp(b'x_i)}{1 + \exp(b'x_i)} = \frac{1}{1 + \exp(-b'x_i)}$$

(logit model)
Estimation credit scores with logit

The first step in MLE is to set up the likelihood function. For the borrower that defaulted:

\[ \text{Prob(Default}_i\text{)} = \text{Prob}(y_i = 1) = \Lambda (\mathbf{b}' \mathbf{x}_i) \]

For the borrower that did not default, we get the likelihood:

\[ \text{Prob(NoDefault}_i\text{)} = \text{Prob}(y_i = 0) = 1 - \Lambda (\mathbf{b}' \mathbf{x}_i) \]

The likelihood for observation i can be written as:

\[ L_i = (\Lambda (\mathbf{b}' \mathbf{x}_i))^{y_i} (1 - \Lambda (\mathbf{b}' \mathbf{x}_i))^{1-y_i} \]

Assuming that defaults are independent, the likelihood of a set of observations is a product of the individual likelihoods:

\[ L = \prod_{i=1}^{N} L_i = \prod_{i=1}^{N} (\Lambda (\mathbf{b}' \mathbf{x}_i))^{y_i} (1 - \Lambda (\mathbf{b}' \mathbf{x}_i))^{1-y_i} \]

And then, logarithm of likelihood is maximized. \textbf{b}-vector is estimated then.
Estimation credit scores with logit

Predictions and assessment of prediction quality:

**ROC curve** - receiver operating characteristic curve is a graphical plot that illustrates the diagnostic ability of a binary classifier system as its discrimination threshold is varied.

TP – true positive
FP – false positive
FN – false negative
TN – true negative

**Estimation credit scores with logit**

**AUC (A)** – Area Under the Curve, proportion of area under the curve relative to the total area of the unit square.

A=0.5 -> random model

A=1.0 -> perfect discrimination

Contingency table

\[
P(\text{FP}) = \text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}}
\]

\[
P(\text{TP}) = \text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}}
\]

More about this topic during next lecture about model validation...
Estimation credit scores with logit

See example:
01_credit_scores_logit.R

**Exercise:**
Estimate once again the logit model but choose only 2 explanatory variables. Use winsorizing techniques to deal with outliers if needed. Compare results with the first version of the model.