

On Foundations of Insurance Mathematics

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History of Insurance in Europe

- Prehistory
- Maritime insurance
- Fire Insurance
- Life Insurance
 - Price of annuities and other types of life insurance
 - Premium calculation
- Pension systems

Prehistory

- Early methods of transferring or distributing risk:
 - Chinese merchants travelling treacherous river rapids would redistribute their wares across many vessels to limit the loss due to any single vessel's capsizing (3rd millenium BC)
 - The Greeks and Romans introduced the origins of health and life insurance c. 600 AD when they organized guilds called "benevolent societies" which cared for the families and paid funeral expenses of members upon death.
 - Guilds in the Middle Ages served a similar purpose

Maritime Insurance

- In the late 1680s, Mr. Edward Lloyd opened a coffee house that became a popular haunt of ship owners, merchants, and ships' captains, and thereby a reliable source of the latest shipping news
- It became the meeting place for parties wishing to insure cargoes and ships, and those willing to underwrite such ventures

Fire Insurance

- Insurance as we know it today can be traced to the Great Fire of London, which in 1666 devoured 13 200 houses
- In the aftermath of this disaster, Nicholas Barbon opened an office to insure buildings. In 1680, he established England's first fire insurance company, "The Fire Office," to insure brick and frame homes

Life Insurance

- The concept of Annuity – a yearly salary paid till one's death was elaborated in early 1690s by a mathematician and astronomer Edmund Halley
- He published in Philosophical Transactions of the Royal Society an article *An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslaw; with an Attempt to ascertain the Price of Annuities upon Lives*

Halley's article

- Halley underlined the importance of the proper study of people's mortality
- We read in his article:

THE Contemplation of the Mortality of Mankind [...] have been [...] considered by [...] Sir Willian Petty, in his Natural and Political Observations on the Bills of Mortality of London [...] But the Deduction from those Bills of Mortality seemed even to their Authors to be defective: First, In that the Number of the People was wanting. Secondly, That the Ages of the People dying was not to be had. And Lastly, That both London and Dublin by reason of the great an casual Accession of Strangers who die therein, (as appeared in both, by the great Excess of the Funerals above the Births) rendered them incapable of being Standards for this purpose

Halley's article, continued

- We further read:
 - This *Defect seems in a great measure to be satisfied by the late curious Tables of the Bills of Mortality at the City of Breslaw [...] wherein both the Ages and Sexes of all that die are monthly delivered, and compared wiht the number of the Births, for Five Years last past, viz. 1687, 88, 89, 90, 91 [...]*
 - This City of *Breslaw is the Capital City of the Province of Silesia; near the Confines of Germany and Poland [...] It is very far from the Sea [...]* confluence of Strangers is but small, and the Manufacture of Linnen employs chiefly the poor People of the place, as well as of the Country round about

Life tables, observations

- I will suppose the People of *Breslaw* to be increased by *1238 Births* annually.
- Of these it appears by the same Tables, that *348* do die *yearly in the first Year of their Age*,
- *and that but 890* do arrive at a full *Years Age*;
- *and likewise, that 198* do die in the *Five Years* between *1* and *6* compleat, taken at a *Medium*;
- *so that but 692* of the *Persons born* do survive *Six whole Years*.

Life tables of Breslaw population - numbers

Age.	Per	Age.	Per	Age.	Per
0	1000	7	680	14	628
1	855	8	670	15	622
2	798	9	661	16	616
3	760	10	653	17	610
4	732	11	646	18	604
5	710	12	640	19	598
6	692	13	634	20	592

Life tables of Breslaw population - numbers

Age.	Per	Age.	Per	Age.	Per
43	419	50	346	57	272
44	409	51	335	58	262
45	397	52	324	59	252
46	387	53	313	60	242
47	377	54	302	61	232
48	367	55	292	62	222
49	357	56	282	63	212

Life tables of Breslaw population - numbers

Age.	Per	Age.	Per	Age.	Per
64	202	71	131	78	58
65	192	72	120	79	49
66	182	73	109	80	41
67	172	74	98	81	34
68	162	75	88	82	28
69	152	76	78	83	23
70	142	77	68	84	20

Calculation of the price of Annuity

- Question: what price shall pay a person of age x who wants to buy a Life Annuity of EUR10000, i.e. yearly payment of EUR10000 till his/her death since the beginning of the next year of his/her life?
- We need to know the EXPECTED VALUE OF THE MONEY PAID FOR SUCH A PERSON

Calculation of the price of Annuity, continued

- Let l_y be the number of persons of the same sex who were among say 1000 persons born y years ago and are still alive at their age of y
- Let i denote the yearly interest rate (i.e. **the present value** of 1 EURO which we will obtain after n years equals $(1+i)^{-n}$)
- The price life annuity equals

$$a_x = 10000 \left((1+i)^{-1} \frac{l_{x+1}}{l_x} + (1+i)^{-2} \frac{l_{x+2}}{l_x} + \dots \right)$$

Other types of insurance

- Finite Horizon Annuity: yearly payment of say EUR10000 since the beginning of the next year of one's life for next n years or till his/her death

- The price of such annuity equals

$$a_{x:n} = 10000 \left((1+i)^{-1} l_{x+1} + (1+i)^{-2} l_{x+2} + \dots + (1+i)^{-n} l_{x+n} \right) / l_x$$

- Life Insurance - single payment of say EUR10000 at the moment of one's death

$$A_{x:\infty}^1 = 10000 \left((1+i)^{-1} (l_{x+1} - l_{x+2}) + (1+i)^{-2} (l_{x+2} - l_{x+3}) + \dots \right) / l_x$$

Premium calculation

- Instead of paying once, annuity and life insurance are may be paid with several payments
- Let us assume that a person at age x wants to pay yearly (since now) a premium P in order to get life annuity of EUR10000 at the age $x+n$. We have

$$P = 10000 \frac{(1+i)^{-n} a_{x+n}}{1 + a_{x:n-1}}$$

Pensions across Europe – history and current regulation

- Great Britain
 - **1670s** First organized pension scheme for Royal Navy Officers
 - **1908** Old Age Pensions Act - introduced first general old age pension paying a non-contributory amount of between 10p and 25p a week, from age 70, on a means-tested basis from January 1 1909 - "Pensions Day". This was introduced during the Liberal government of David Lloyd-George. Sir William Beveridge, father of the welfare state, was an adviser.

Pensions across Europe – history and current regulation

- **1925** Contributory Pensions Act - set up a contributory State scheme for manual workers and others earning up to £250 a year. The pension was 50p a week from age 65.
- **1946** National Insurance Act - introduced contributory State pension for all. Initially pensions were £1.30 a week for a single person and £2.10 for a married couple. Paid from age 65 for men and 60 for women, effective from 1948.

Pensions across Europe – history and current regulation

- **1959** National Insurance Act - introduced a top-up state pensions scheme, based on earnings and known as the graduated pension. Covered earnings between £9 and £15 a week.
- **1975** Social Security Pensions Act - set up the State Earnings related Pension Scheme (Serps). Introduced in 1978, the scheme replaced graduated pensions. Rules for contracting out were also introduced, whereby workers with adequate private provision can give up all or part of the benefits of Serps. In return they pay lower National Insurance contributions.

Pensions across Europe – history and current regulation

- **1980** Social Security Act - Link between state pension increases and average earnings broken by Margaret Thatcher's Conservative government. If the link with earnings had not been broken, a basic state pension for a single pensioner would worth about £30 a week more.
- **1986** Financial Services Act - set out terms and conditions under which investment business could be conducted. Changes to contracting out. Taxation of pension fund "surpluses" introduced.

Pensions across Europe – history and current regulation

- Germany
 - Bismarck advised Emperor Wilhelm II to establish social Insurance
 - **1891** old-age pension plans if one was 70 years old (average life expectancy was 45 years at the time)
 - **1911** The Imperial Insurance Code signaled the next step towards a modern pension scheme. From that point on, it was also for example lawfully provided that the bereaved were looked after by the state if they could not sustain themselves after the death of their spouse

Pensions across Europe – history and current regulation

- **1957** it was here that the pay-as-you-go scheme that is so typical for Germany was introduced, as well as a pension formula that calculates the earnings during old age based on the obtained earnings during the time in gainful employment
- **1970s** the retirement age was shaped more flexibly and now, self-employed people, students and housewives could profit of the lawful pension insurance.

Problems

- Aging society ...

Insurance market worldwide

- Insurance premiums worldwide (in mln EUR)

Country	1991	2001	2003
USA	421 715	1 061 349	980 885
EU	327 993	779 620	819 291
Japan	249294	498 452	421 401
Rest of the World	146 625	352 990	364 878

Source: World Insurance in 2003, Swiss Re

Life insurance – about 40%-60% in EU

Non life insurance (e.g. car insurance)