Advanced Macroeconomics

The Ramsey Model

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- Authors: Frank Ramsey (1928), David Cass (1965) and Tjalling Koopmans (1965)
- Basically the Solow model with endogenous savings - explicit consumer optimization
- Probably the most important model in contemporaneous macroeconomics, workhorse for many areas, including business cycle theories

Basic setup

- Closed economy
- No government
- One homogeneous final good
- Price of the final good normalized to 1 in each period (all variables expressed in real terms)
- Two types of agents in the economy:
  - Firms
  - Households
- Firms and households identical: one can focus on a representative firm and a representative household, aggregation straightforward

Firms

- Final output produced by competitive firms
- Neoclassical production function with Harrod neutral technological progress

$$Y_t = F(K_t, A_t L_t) \tag{1}$$

- Capital and labour inputs rented from households
- Technology is available for free and grows at a constant rate $g > 0$:

$$A_{t+1} = (1+g)A_t$$

- Maximization problem of firms:

$$\max_{L_t, K_t} \{F(K_t, A_t L_t) - W_t L_t - R_{K,t} K_t\}$$

- Firms maximize their profits, taking factor prices as given (competitive factor markets)
- First order conditions:

$$W_t = \frac{\partial F}{\partial L_t} \qquad R_{K,t} = \frac{\partial F}{\partial K_t} = f'(k_t) \tag{2}$$

- Own production factors (capital and labour), so earn income on renting them to firms
- Labour supplied inelastically, grows at a constant rate $n > 0$:

$$L_{t+1} = (1+n)L_t$$

- Capital is accumulated from investment I_t and subject to depreciation:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

- Total income of households can be split between consumption or savings (equal to investment):

$$W_t L_t + R_{K,t} K_t = C_t + S_t = C_t + I_t \tag{4}$$

- Make optimal consumption-savings decisions

Households II

- Households maximize the lifetime utility of their members (present and future):

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(\tilde{C}_t) L_t \tag{5}$$

where:

- $\tilde{C}_t = \frac{C_t}{L_t}$ - consumption per capita
  - β - discount factor $(0 < \beta < 1)$
  - $u(C_t)$ - instantaneous utility from consumption:

$$u(\tilde{C}_t) = \frac{\tilde{C}_t^{\,1-\theta}}{1-\theta} \tag{6}$$

  where:

    - $\theta > 0$
    - If $\theta = 1$ then $u(\tilde{C}_t) = \ln \tilde{C}_t$

Households III

- Remarks:
- Literally: household members live forever
  - Justification: intergenerational transfers, people care about utility of their offspring
  - Discounting: households are impatient
- Remarks on the utility function:
  - $u(\tilde{C}_t)$ is a constant relative risk aversion function (CRRA):

$$-\frac{\tilde{C}_t u''(\tilde{C}_t)}{u'(\tilde{C}_t)} = \theta$$

  - $u(\tilde{C}_t)$ is a constant intertemporal elasticity of substitution function:

$$-\frac{\partial \ln\left(\frac{\tilde{C}_1}{\tilde{C}_2}\right)}{\partial \ln\left(\frac{u'(\tilde{C}_1)}{u'(\tilde{C}_2)}\right)} = \frac{1}{\theta}$$

  - CRRA form essential for balanced growth (steady state)

Households IV

- Households' optimization problem: maximize (5) subject to the model's constraints:
- Capital law of motion (capital is the only asset held by households), incorporating income definition and savings-investment equality (4)
  - Transversality condition:

$$\lim_{t\to\infty}\left(K_{t+1}\prod_{s=1}^{t}\frac{1}{1+r_s}\right)=0 \tag{7}$$

  where: $r_t = R_{K,t} - \delta = f'(k_t) - \delta$ is the market rate of return on capital (real interest rate)

- Interpretation of the transversality condition:
  - Analog of a terminal condition in a finite horizon
  - Non-negativity constraint on the terminal (net present) value of assets held by households (capital)
  - No-Ponzi game condition: proper lifetime budget constraint on households
- Optimization by households implies (7) holds with equality

General equilibrium

- Market clearing conditions:
- Output produced by firms must be equal to households' total spending (on consumption and investment):

$$Y_t = C_t + I_t \tag{8}$$

  - Labour supplied by households must be equal to labour input demanded by firms
  - Capital supplied by households must be equal to capital input demanded by firms

Definition of competitive equilibrium

A sequence of $\{K_t, Y_t, C_t, I_t, W_t, R_{K,t}\}_{t=0}^{\infty}$ for a given sequence of $\{L_t, A_t\}_{t=0}^{\infty}$ and an initial capital stock K_0, such that (i) the representative household maximizes its utility taking the time path of factor prices $\{W_t, R_{K,t}\}_{t=0}^{\infty}$ as given; (ii) firms maximize profits taking the time path of factor prices as given; (iii) factor prices are such that all markets clear.

Households' optimization problem

- Lifetime utility rewritten:

$$
\begin{aligned}
U_0 &= \sum_{t=0}^{\infty} \beta^t \frac{\tilde{C}_t^{\,1-\theta}}{1-\theta} L_t = \\
&= L_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\tilde{C}_t^{\,1-\theta}}{1-\theta} \qquad (9)
\end{aligned}
$$

where:

$$
\tilde{\beta} = \beta(1+n) \qquad \beta(1+g)^{1-\theta}(1+n) < 1
$$

the last inequality is assumed and ensures that utility is bounded

Households' optimization problem II

- Capital accumulation rewritten in per capita terms (using (4)):

$$\tilde{K}_{t+1} = \frac{1-\delta}{1+n}\tilde{K}_t + \frac{1}{1+n}(W_t + R_{K,t}\tilde{K}_t - \tilde{C}_t) \quad (10)$$

- Capital accumulation rewritten in intensive form (using (4) and defining $w_t = \frac{W_t}{A_t}$):

$$k_{t+1} = \frac{1-\delta}{(1+g)(1+n)}k_t + \frac{1}{(1+g)(1+n)}(w_t + R_{K,t}k_t - c_t) \quad (11)$$

- Transversality condition rewritten in intensive form:

$$\lim_{t\to\infty}\left(k_{t+1}\prod_{s=1}^{t}\frac{(1+n)(1+g)}{1+r_s}\right) = 0 \quad (12)$$

Lagrange function and FOCs

- Lagrange function (normalizing L_0):

$$LL = \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{\tilde{C}_t^{1-\theta}}{1-\theta} + \lambda_t \left[\begin{array}{c} (1-\delta)\tilde{K}_t + W_t + R_{K,t}\tilde{K}_t - \tilde{C}_t - \\ -(1+n)\tilde{K}_{t+1} \end{array} \right] \right)$$

- First order conditions (FOCs):

$$\frac{\partial LL}{\partial \tilde{C}_t} = 0 \implies \tilde{C}_t^{-\theta} = \lambda_t \tag{13}$$

$$\frac{\partial LL}{\partial \tilde{K}_{t+1}} = 0 \implies \tilde{\beta}\lambda_{t+1}\left(1-\delta+R_{K,t+1}\right)) = (1+n)\lambda_t \tag{14}$$

Euler equation I

- Equations (13) and (14) imply:

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta} = \tilde{\beta}\frac{R_{K,t+1}+1-\delta}{(1+n)} \tag{15}$$

- Using the definition of $\tilde{\beta}$ and rewriting in intensive form:

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta\frac{R_{K,t+1}+1-\delta}{(1+g)^{\theta}} \tag{16}$$

- For consumption per capita, using also the definition of the interest rate r_t:

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta} = \left(\frac{c_{t+1}A_{t+1}}{c_tA_t}\right)^{\theta} = \beta\left(1+r_{t+1}\right) \tag{17}$$

Euler equation II

- Interpretation of the Euler equation (17):
- For $\theta > 0$: $\tilde{C}_{t+1} > \tilde{C}_t \iff 1 + r_{t+1} > \beta^{-1}$
  - Interpretation: For (per capita) consumption to grow the (market) interest rate must exceed households' rate of time preference
  - Interpretation: It is optimal for households to postpone consumption (i.e. save in the current period and consume more in the next period) iff the related utility loss is more than offset by the rate of return on savings
- Role of θ:
  - The higher θ the less responsive consumption to changes in the interest rate
  - In other words: The higher θ the stronger the consumption smoothing motive (the lower intertemporal substitution)

Equilibrium dynamics in intensive form - summary

- The equilibrium dynamics of the model at any time t can be characterized by 3 equations: (16), (11) and (12). They are (after some rewriting and using (4) and (8) in intensive form):

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta \frac{f'(k_{t+1}) + 1 - \delta}{(1+g)^{\theta}} \tag{18}$$

$$\frac{k_{t+1}}{k_t} = \frac{1-\delta}{(1+g)(1+n)} + \frac{1}{(1+g)(1+n)} \frac{f(k_t) - c_t}{k_t} \tag{19}$$

$$\lim_{t\to\infty} \left(k_{t+1} \prod_{s=1}^{t} \frac{(1+n)(1+g)}{f'(k_{t+1}) + 1 - \delta} \right) = 0 \tag{20}$$

- At $t = 0$ capital is fixed. For given initial k_0 and c_0, equations (18) and (19) describe the future evolution of these variables: k_t and c_t. The transversality condition (20) pins down the initial level of consumption c_0.

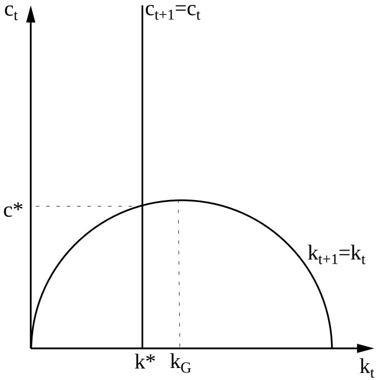
Steady state equilibrium I

- In the steady state equilibrium k_t and c_t must be constant.
- Using (18) and (19), the long-run solution to the Ramsey model:

$$f'(k^*) = \frac{(1+g)^\theta}{\beta} - 1 + \delta \tag{21}$$

$$c^* = f(k^*) - (n + g + \delta + ng)k^* \tag{22}$$

- Plotting (21) and (22) in the (k, c) space:



Modified golden rule I

- How do we know that $k^* < k_G$?
- From (22):

$$f'(k_G) = n + g + \delta + ng$$

  - The transversality condition (20) written in the steady-state:

$$\lim_{t\to\infty} k^* \left(\frac{(1+n)(1+g)}{f'(k^*)+1-\delta}\right)^t = 0$$

  - This implies:

$$f'(k^*) > n + g + \delta + ng$$

  - Since $f''(k) < 0$ for any $k > 0$

$$f'(k^*) > f'(k_G) \implies k^* < k_G \tag{23}$$

Modified golden rule II

- Equivalently, we can show that the steady-state savings rate s^* falls short of the savings rate consistent with the golden rule:
- From (22), the steady-state savings rate is:

$$s^* = 1 - \frac{c^*}{f(k^*)} = (n + g + \delta + ng)\frac{k^*}{f(k^*)}$$

  - Using (23):

$$s^* < f'(k^*)\frac{k^*}{f(k^*)} = \alpha(k^*)$$

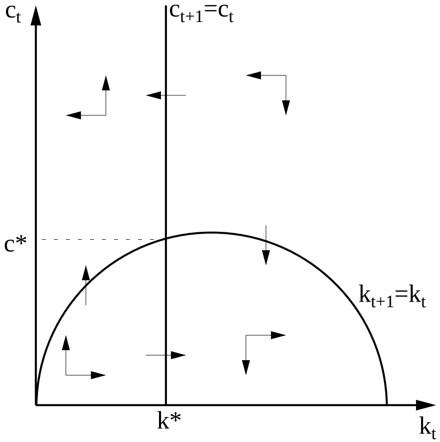
- Intuitive explanation: households are impatient ($\beta < 1$) and smooth consumption ($\theta > 0$, relevant if $g > 0$).

The role of the discount factor

- Higher β implies more patient consumers
- From (21): if β goes up, $f'(k^*)$ goes down, which means that k^* goes up
- The $c_{t+1} = c_t$ locus on the (k, c) chart shifts right
- Steady-state consumption goes up
- Intuition: if households are more patient, they save more, which brings them closer to the standard golden rule

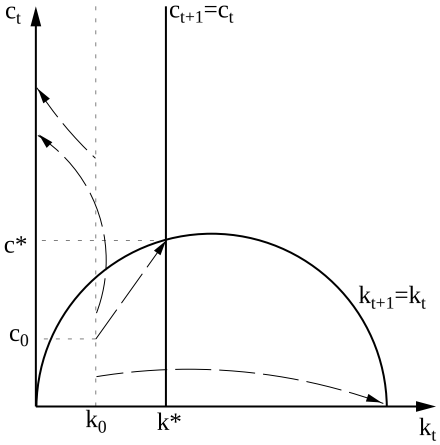
Phase diagram

- From (18): $k \lesseqgtr k^* \implies \Delta c \gtreqless 0$
- From (19): $c \lesseqgtr f(k) - (n + g + \delta + ng)k \implies \Delta k \gtreqless 0$



Saddle path (stable arm)

- Transversality condition (20) pins down the inital level of c_0 for any initial k_0, so that the system converges to the steady-state:



Uniqueness of equilibrium

- How do we know that the saddle path is a unique equilibrium?
- If the initial level of consumption were below c_0:
  - capital would eventually reach its maximal level $\bar{k} > k_G$
    - this implies:
$$f'(\bar{k}) < f'(k_G) = n + g + \delta + ng$$
    - which violates the transversality condition (20) since:
$$\lim_{t\to\infty} \bar{k} \left(\frac{(1+n)(1+g)}{f'(\bar{k}) + 1 - \delta} \right)^t = \infty$$
    - informally (but more intuitively): at the end of their planning horizon, households would hold very valuable assets, which cannot be optimal
  - If the initial level of consumption were above c_0:
    - capital would eventually reach 0 but consumption would stay positive, which is clearly not feasible

Speed of convergence

- Compared to the Solow model, the speed of convergence in the Ramsey model depends additionally on the behaviour of the savings rate along the transition path
- For very small time intervals, the following implications hold (see Barro and Sala-i-Martin, 2004, ch. 2.6.4):
  - $\frac{1}{\theta} < s^* \implies s_t - s^*$ depends positively on $k_t - k^*$
  - $\frac{1}{\theta} = s^* \implies s_t = s^*$
  - $\frac{1}{\theta} > s^* \implies s_t - s^*$ depends negatively on $k_t - k^*$
- Intuition (suppose the economy starts from $k_0 < k^*$, so $c_0 < c^*$):
  - if households care much about consumption smoothing (θ is high), they wll try to shift consumption from the future to the present
  - if households care little about consumption smoothing (θ is low), they will try to postpone consumption to reach steady-state sooner
- For standard parameter values $\frac{1}{\theta} > s^*$, so the Ramsey model predicts relatively fast pace of convergence

Wealth and consumption

- Consider the lifetime budget constraint of the household

$$C_0 + \sum_{t=1}^{\infty} \frac{C_t}{R_t} = (1 + r_0)K_0 + W_0 L_0 + \sum_{t=1}^{\infty} \frac{W_t L_t}{R_t} \equiv \Omega_t$$

where Ω_t is lifetime wealth. Substitute recursively from Euler

$$\frac{C_{t+1}}{C_t} = \left(\beta\left(1 + r_t\right)\right)^{\frac{1}{\theta}}\left(1 + n\right)$$

$$C_0 + C_0 \frac{\left(\beta\left(1 + r_1\right)\right)^{\frac{1}{\theta}}\left(1 + n\right)}{R_1} + C_0 \frac{\left(\beta^2\left(1 + r_1\right)\left(1 + r_2\right)\right)^{\frac{1}{\theta}}\left(1 + n\right)^2}{R_2} + \ldots = \Omega_t$$

Wealth and consumption cont'd

- so that

$$C_0 = \Omega_t \left[1 + \sum_{t=1}^{\infty} \frac{\left(\beta^t R_t\right)^{\frac{1}{\theta}} (1+n)^t}{R_t} \right]^{-1}$$

- For log utility this boils down to

$$C_0 = \left[1 - \beta \left(1+n\right)\right] \Omega_t$$

- So current consumption depends on lifetime wealth. Current income has a small impact on current consumption!

Efficiency of equilibrium

- A very important question in economics: are markets efficient?
- What does it mean? If markets are efficient the equilibrium allocation maximizes households welfare.
- Then government intervention is pointless (see Adam Smith's invisible hand)
- But we know many situations where markest fail. E.g.:
  - externalities
  - imperfect information
  - principal-agent problem
- How about the Ramsey model?

Social planer's problem

- Imagine a social planer - allocates resources and wishes to maximize household welfare
- Constrained by technology and available resources
- For simplicity consider a model without labor or technology growth

$$\max_{c_t, k_{t+1}, y_t} U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{24}$$

subject to

$$y_t = f(k_t)$$

and

$$c_t + k_{t+1} = y_t + (1-\delta)k_t$$

Lagrangean and FOC

$$L = \sum_{t=0}^{\infty} \beta^t u(c_t) - \sum_{t=0}^{\infty} \lambda_t \left[c_t + k_{t+1} - f(k_t) - (1-\delta) k_t \right] \tag{25}$$

FOC:

$$c_t \quad : \quad \beta^t u_{c,t} = \lambda_t$$

$$k_{t+1} \quad : \quad -\lambda_t + \lambda_{t+1} \left[f'(k_{t+1}) + 1 - \delta \right] = 0$$

$$TVC \quad (imposed) : \quad \lim_{t \to \infty} \lambda_t k_{t+1} = 0$$

Optimal allocation

- The optimal alocation is given by

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = f'(k_{t+1}) + 1 - \delta$$

$$TVC \quad : \quad \lim_{t \to \infty} \beta^t u_{c,t} k_{t+1} = 0$$

$$y_t = f(k_t)$$

$$c_t + k_{t+1} = y_t - (1 - \delta) k_t$$

Comparison to decentralized (competitive) equilibrium

- The competitive equilibrium is given by (16), (11) and (12), which after ignoring labor and population growth become:

$$\frac{u_{c,t}}{u_{c,t+1}} = \beta \left(R_{K,t+1} + 1 - \delta \right)$$

$$k_{t+1} = (1 - \delta) k_t + (w_t + R_{K,t} k_t - c_t)$$

$$\lim_{t \to \infty} \left(k_{t+1} \prod_{s=1}^{t} \frac{1}{1 + r_s} \right) = 0$$

Competitive equilibrium cont'd

- Use zero profit condition:

$$k_{t+1} = (1-\delta)\, k_t + y_t - c_t$$

- Note that

$$\frac{u_{c,t+1}}{u_{c,t}} = \beta(1+r_{t+1})$$

- so that

$$\prod_{s=1}^{t} \frac{1}{1+r_s} = \frac{\beta u_{c,1}}{u_{c,0}} \frac{\beta u_{c,2}}{u_{c,1}} \cdots \frac{\beta u_{c,t}}{u_{c,t-1}} = \frac{\beta^t u_{c,t}}{u_{c,0}}$$

- Hence TVC condition is equivalent to

$$\lim_{t\to\infty} \left(\beta^t u_{c,t} k_{t+1}\right) = 0$$

Welfare theorems

- Hence, social planner's and competitive allocations are equal. Ramsey economy reflects two fundamental welfare theorems.

1. First welfare theorem: every competitive equilibrium is efficient.
2. Second welfare theorem: any efficient allocation can be supported as a competitive equilibrium (possibly with lump-sum transfers).

- First states that resources are not wasted in a competitve economy (Adam Smith would like it).
- Second gives the equivalence beween an efficient and competitive allocation and thus, the possibility to solve the (simpler) planners problem.

The government

- In modern economies governments are big players
- $\frac{G}{Y}$ is 40-50% (once all government expenditure is accounted for)
- Some problems worth analysing:
  - difference/ similarity between tax and debt financing of gov. expenditure
  - reaction of economy to government shocks (spending, taxes)

Ricardian equivalence

- In the Ramsey model Ricardian equivalence holds:
- Government plans sustainable - initial debt plus net present value of expenditures equals net present value of revenues

$$T_0 + \sum_{t=1}^{\infty} \frac{T_t}{R_t} = (1 + r_0)B_0 + G_0 + \sum_{t=1}^{\infty} \frac{G_t}{R_t}$$

- where

$$R_t \equiv \prod_{i=1}^{t} (1 + r_i)$$

Ricardian equivalence cont'd

- Household lifetime budget (with government)

$$C_0 + \sum_{t=1}^{\infty} \frac{C_t}{R_t} = (1 + r_0)K_0 + (1 + r_0)B_0 + W_0 L_0 + \sum_{t=1}^{\infty} \frac{W_t L_t}{R_t} - T_0 - \sum_{t=1}^{\infty} \frac{T_t}{R_t}$$

- Substitute from government budget for $\sum_{t=1}^{\infty} \frac{T_t}{R_t}$

$$C_0 + \sum_{t=1}^{\infty} \frac{C_t}{R_t} = (1 + r_0)K_0 + W_0 L_0 + \sum_{t=1}^{\infty} \frac{W_t L_t}{R_t} - G_0 - \sum_{t=1}^{\infty} \frac{G_t}{R_t}$$

- What matters for the household is the present value of expenditures
- Financing expenditures with lump-sum taxes equivalent to financing expenditures with debt

Ricardian equivalence in practice

- Empirical research shows that Ricardian equivalence holds partly
- Why? Some explanations can be given within our modeling framework.
  - finite horizon of households
  - different interest rates
  - credit constrained households
  - distortionary taxes
- Good to know the theoretical benchmark where Ricardian equivalence holds
- For instance to be able to construct models where gov. deficits have macro effects

Distortionary taxation

- The government is assumed to run a balanced budget each period:

$$G_t = V_t + \tau_w W_t L_t + \tau_k (R_{K,t} - \delta) K_t + \tau_c C_t + \tau_i I_t + \tau_f (Y_t - W_t L_t - \delta K_t) \tag{26}$$

where:

- G_t - government purchases (exogenous)
- τ_w, τ_k, τ_c, τ_i, τ_f - proportional tax rates on wage income, capital income, consumption, investment and firms' taxable profits, respectively (all exogenous)
- V_t - lump-sum taxes (net of lump-sum transfers) from households, adjusted so that the balanced budget constraint (26) holds

Modified firms' problem

- Maximization problem of firms:

$$\max_{L_t, K_t} \{F(K_t, A_t L_t) - W_t L_t - R_{K,t} K_t - \tau_f (F(K_t, A_t L_t) - W_t L_t - \delta K_t)\}$$

- First order conditions (using definition $r_t = R_{K,t} - \delta$):

$$W_t = \frac{\partial F}{\partial L_t} \qquad \frac{r_t}{1 - \tau_f} + \delta = \frac{\partial F}{\partial K_t} = f'(k_t)$$

- Firms are competitive so earn zero profits:

$$Y_t = W_t L_t + R_{K,t} K_t + \tau_f (Y_t - W_t L_t - \delta K_t) \tag{27}$$

- Market clearing on the product market:

$$Y_t = C_t + I_t + G_t \tag{28}$$

Modified households' problem

- We assume that government actions do not affect utility directly, so households' lifetime utility is still given by (5)
- Households' budget constraint (4) becomes:

$$W_t L_t + R_{K,t} K_t - \tau_w W_t L_t - \tau_k \left(R_{K,t} - \delta\right) K_t - V_t = (1+\tau_c) C_t + (1+\tau_i) I_t \tag{29}$$

- Note that, by (27), households' factor income is no longer equal to output
- Modified transversality condition:

$$\lim_{t \to \infty} \left(K_{t+1} \prod_{s=1}^{t} \frac{1}{1 + (1 - \tau_k) r_s} \right) = 0 \tag{30}$$

Modified households' optimization problem

- Lifetime utility (identical to (9)):

$$U_0 = L_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\tilde{C}_t^{1-\theta}}{1-\theta} \tag{31}$$

- Capital accumulation (substituting for investment from (29)):

$$\begin{aligned}\tilde{K}_{t+1} &= \frac{1-\delta}{1+g}\tilde{K}_t + \frac{1}{(1+g)(1+\tau_i)} \\ &\quad \left[\begin{array}{c} (1-\tau_w)W_t + (1-\tau_k)R_{K,t}\tilde{K}_t + \tau_k\delta\tilde{K}_t \\ -(1+\tau_c)\tilde{C}_t - \tilde{V}_t \end{array} \right]\end{aligned} \tag{32}$$

- Transversality condition:

$$\lim_{t\to\infty}\left(\tilde{K}_{t+1}\prod_{s=1}^{t}\frac{1+n}{1+(1-\tau_k)r_s}\right) = 0 \tag{33}$$

Equilibrium dynamics

- The equilibrium dynamics of the model at any time t is given by the following equations:
- Euler equation (maximizing (31), subject to (32) and using firms' FOC):

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta \frac{1+\tau_i(1-\delta)+(1-\tau_k)(1-\tau_f)(f'(k_{t+1})-\delta)}{(1+g)^{\theta}(1+\tau_i)} \quad (34)$$

  - Capital accumulation equation (32) (merged with government budget constraint (26) and firms' zero profit condition (27), with $g_t = \frac{G_t}{A_t L_t}$):

$$\frac{k_{t+1}}{k_t} = \frac{1-\delta}{(1+g)(1+n)} + \frac{1}{(1+g)(1+n)} \frac{f(k_t)-c_t-g_t}{k_t} \quad (35)$$

  - Transversality condition (33):

$$\lim_{t\to\infty}\left(k_{t+1}\prod_{s=1}^{t}\frac{(1+n)(1+g)}{f'(k_{t+1})+1-\delta}\right)=0 \quad (36)$$

Simplified variant - only capital tax

- Euler equation becomes:

$$\left(\frac{c_{t+1}}{c_t}\right)^\theta = \beta\frac{1+(1-\tau_k)(f'(k_{t+1})-\delta)}{(1+g)^\theta}$$

  Capital accumulation equation:

$$\frac{k_{t+1}}{k_t} = \frac{1-\delta}{(1+g)(1+n)} + \frac{1}{(1+g)(1+n)}\frac{f(k_t)-c_t-g_t}{k_t}$$

- Steady state:

$$f'(k^*) = \frac{(1+g)^\theta-\beta}{\beta(1-\tau_k)}+\delta$$
$$c^* = f(k^*)-(n+g+\delta+ng)k^*-g^*$$

Simplified variant - simulate shocks

- Permanent increase in government expenditure (financed with lump-sum taxes)
- Temporary increase in government expenditure (financed with lump-sum taxes)
- Permanent increase in capital tax

Conclusions from model with government

- In the Ramsey model:
  - Ricardian equivalence holds as long as taxes are nondistortionary
  - Taxes on capital, investment and firm profits are distortionary (change the equilibrium allocation)
  - Taxes on consumption and labor income are nondistortionary (but only as long as labor supply is exogeneous)
  - Government expenditure crowds out private consumption one-to-one but does not affect capital or output (if financed by nondistortionary taxes)

Some simulations with the Ramsey model

- Several interesting numerical simulations can be done with the Ramsey model
  - find initial consumption for given k_0 that fulfils the equilibrium conditions
  - simulate convergence from c_0, k_0 to steady state
  - simulate a change (temporary or permanent) of:
    - rate of time preference
    - government expenditure
    - capital tax

Main implications of the Ramsey model

- As in the Solow model, long-run growth (of output per capita) possible only with technological progress (exogenous in both models)
- We should observe conditional, but not necessarily unconditional, convergence (in line with the data)
- Shows several important (though ratehr benchmark) features
  - Ricardian equivalence
  - Key role of permanent income (lifetime wealth) in driving consuption
- Compared to the Solow model:
  - Explicit optimality criterion - households' utility
  - If there is no distortionary taxation (i.e. if there is no government or all taxes are lump-sum), allocations are Pareto optimal: decentralized equilibrium coincides with allocations dictated by a benevolent social planer (markets are competitive and complete, so the first welfare theorem applies)
  - Savings rate endogenous and in the long-run always lower than implied by the golden rule