Theorem 1 (Minkowski) Let \( A, B \) be two convex sets in \( \mathbb{R}^L \) and \( A \cap B = \emptyset \). Then there exists \( p \in \mathbb{R}^L, p \neq 0 \) and a number \( z \in \mathbb{R} \) such that, \( p \cdot x \geq z \geq p \cdot y \) for any \( x \in A, y \in B \).

Consider a maximization problem \( \max_{x \in X} f(x) \) constrained by \( g_k(x) \leq w_k \) for all \( k = 1, \ldots, K \). We say that \( x \in X \) is feasible, if \( g_k(x) \leq w_k \) for all \( k = 1, \ldots, K \).

Theorem 2 (Kuhn-Tucker) Let \( X \subset \mathbb{R}^N \) be convex, \( f : X \to (\mathbb{R}, \infty) \) concave and \( g_k : X \to (\mathbb{R}, \infty) \) convex. Let \( x^* \) be feasible and there exists nonnegative numbers \( \lambda_1, \ldots, \lambda_K \) such that:

1. for any \( k \lambda_k = 0 \), where \( g_k(x^*) < w_k \),
2. \( x^* \) solves: \( \max_{x \in X} \{ f(x) - \sum_{k=1}^K \lambda_k g_k(x) \} \),

then \( x^* \) solves the maximization problem.

Theorem 3 (Brouwer) Let \( A \subset \mathbb{R}^L \) be non-empty, compact and convex set and function \( f : A \to A \) continuous. Then there exists \( x^* \in A \), such that \( x^* = f(x^*) \).

Theorem 4 (On local invertability of a differentiable function) Let \( f : U \to \mathbb{R}^L \), where \( U \subset \mathbb{R}^n \) is open, be continuously differentiable on \( B_r(x_0) \subset U \) and \( \det f'(x_0) \neq 0 \). Then there exists a neighbourhood \( O = B_\epsilon(x_0) \) \( (\epsilon < r) \), such that function \( f|_O : O \to V \), \( (gdzie f(O) = V) \) is invertible.

Definition 1 (Upper-hemi continuous correspondence) A correspondence \( f : A \to a^Y \), where \( A \subset \mathbb{R}^L \), and \( Y \subset \mathbb{R}^K \) is closed is called upper-hemi continuous, if it has a closed graph and images of compact sets are bounded.

Definition 2 (Lower-hemi continuous correspondence) A correspondence \( f : A \to 2^Y \), where \( A \subset \mathbb{R}^L \), and \( Y \subset \mathbb{R}^K \) is compact, is called lower-hemi continuous, if for any sequence (elements of \( A \)) \( x_m \to x \in A \) and any \( y \in f(x) \), there exists a sequence \( y_m \to y \) and number \( M \) such that \( y_m \in f(x_m) \) for \( m > M \).

Theorem 5 (Berge maximum theorem) Let \( f : Y \to \mathbb{R} \) be continuous, and correspondence \( \Gamma : X \to 2^Y \) continuous\(^1\). If \( \Gamma(x) \neq \emptyset \), then function \( M(x) = \max \{ f(y) : y \in \Gamma(x) \} \) is continuous on \( X \), and correspondence \( \Phi(x) = \{ y \in \Gamma(x) : f(y) = M(x) \} \) is upper-hemi continuous.

\(^1\)I.e. both upper- and lower-hemiconnontinuous.
Theorem 6 (Kakutani) Let $A \subset \mathbb{R}^L$ be a nonempty, compact and convex set and a correspondence $f : A \to 2^A$ upper-hemi continuous. If $f(x)$ is non-empty and convex for every $x \in A$, then there exists $x^* \in A$ such that $x^* \in f(x^*)$.

Theorem 7 (Shapley-Folkman) Let $x \in \text{con}(\sum_{i=1}^L A_i)$, where $(\forall i)A_i \subseteq \mathbb{R}^L$. Then, there exists $(a_i)_{i=1}^L$, such that $x = \sum_{i=1}^L a_i$, and $(\forall i)a_i \in \text{con}(A_i)$ with $a_i \in A_i$ for all but $L$ indexes $i$. 